Optimal Periodic Sensor Scheduling With Limited Resources

Ling Shi, Peng Cheng, and Jiming Chen

***Abstract*—In this technical note, we consider the problem of periodic sensor scheduling with limited resources. Two sensors are used to measure the state of a discrete-time linear process. We assume that each sensor has a maximum duty cycle and at most one sensor can communicate with a remote estimator at each time step due to the limited communication bandwidth. When a sensor is scheduled to send data, it sends the most recent D measurement data to the estimator. Upon receiving the measurement data from the sensors, the estimator computes the optimal estimate of the state of the process. We first present a necessary condition for a periodic sensor schedule to be optimal. Based on this necessary condition, we construct an optimal periodic schedule that minimizes the estimation error at the estimator and at the same time satisfies the energy and communication bandwidth constraints. Examples are provided throughout the technical note to demonstrate the results developed.**

***Index Terms*—Networked control systems (NCSs).**

I. Introduction

Networked control systems (NCSs) have gained much interest in the past decade thanks to the recent advances in fabrication, modern sensor and communication technologies, and computer architectures. Applications of NCSs are found in a growing number of areas, including automobiles, autonomous vehicles, environment and habitat monitoring, industrial automation, power distribution, space exploration, surveillance, transportation, etc [1].

Compared with classic feedback control systems, control over networks can reduce the system wiring, make the system easy to operate, maintain and diagnose, and increase system agility. However, new problems also arise when sensor information and control information flow over a network. For example, due to the limited bandwidth of the network or limited communication energy, data packets may be dropped or delayed, which affects the stability of closed-loop system.

In this technical note, we consider a sensor scheduling problem. In typical NCSs, resources are often limited. For example, sensors may be battery-powered, and the energy for data collection and transmission is limited. Communication channel may be shared by many sensors, and a sensor may not transmit its measurement all the time. Therefore proper scheduling of the sensor data transmission is needed.

Sensor scheduling has been a hot topic of research for many years. Different formulations and approaches have been proposed. Baras and Bensoussan [2] studied nonlinear state estimation problem and considered scheduling a set of sensors so as to optimally estimate a function of an underlying parameter. Walsh *et al.* [3], [4] studied the problem of when to schedule which process to access to the network so that all processes remain stable. Gupta *et al.* [5] considered a different scheduling problem where there are one process and multiple sensors. They proposed a stochastic sensor schedule and provided the optimal probability distribution over the sensors to be selected. Hovareshti *et al.* [6] considered sensor scheduling using smart sensors, i.e., sensors with some memory and processing capabilities, and demonstrated that estimation performance can be improved. Sandberg *et al.* [7] considered estimation over a heterogeneous sensor network. Two types of sensors were investigated: the first type has low-quality measurement but with small processing delay, while the second type has high-quality measurement but with large processing delay. Using a time-periodic Kalman filter, they showed how to find an optimal schedule of the sensor communication. Similar work has been done by Arai *et al.* [8], [9] where fast sensor scheduling was proposed for networked sensor systems. Savage and La Scala [10] considered the problem of optimal sensor scheduling for scalar systems that minimizes the terminal error.

The main contribution of this technical note and comparison with existing work from literature are summarized as follows.

1) In this technical note, we develop schedules of two sensors that can provide the best estimation quality subject to both sensor energy and communication bandwidth constraints. To the best of our knowledge, the framework is novel.
2) Since the solution space contains infinitely many possible schedules which are discrete in nature, find an optimal schedule is in general difficult and challenging, and usually NP hard. As a result, most existing work proposed algorithms and heuristics that typically generate a suboptimal schedule, and nothing in general is said on the optimality of the proposed schedule. However, with some minor assumptions, we are able to provide an optimal periodic schedule that minimizes the estimation error at the estimator and at the same time satisfies the energy and communication bandwidth constraints.

The rest of the technical note is organized as follows. In Section II, we provide the mathematical models of the systems and state the main problem of interest. In Section III, we provide some preliminary results on the state estimation. A necessary condition for a schedule to be optimal is presented in Section IV. Based on this necessary condition, an optimal periodic schedule is constructed in Section V. Examples are provided throughout the technical note to demonstrate the results developed. Some concluding remarks are given at the end.

The following terms that are frequently used in subsequent sections are defined below. $\mathbb{Z}$ is the set of all integers. $\mathbb{S}_+^n$ is the set of n by n positive semidefinite matrices. When $X \in \mathbb{S}_+^n$, we simply write $X \geq 0$; when X is positive definite, we write $X > 0$. $\mathrm{Tr}(X)$ is the trace of X. Let $X \in \mathbb{S}_+^n$ and $Y \in \mathbb{S}_+^n$. We say $X \leq Y$ if $Y - X \geq 0$. Clearly if $Y - X \geq 0$, then $\mathrm{Tr}(Y - X) \geq 0$. For functions $f, f_1, f_2 : \mathbb{S}_+^n \to \mathbb{S}_+^n$, $f_1 \circ f_2$ is defined as $f_1 \circ f_2(X) \triangleq f_1\Big(f_2(X)\Big)$ and f^t is defined as $f^t(X) \triangleq \underbrace{f \circ f \circ \cdots \circ f}_{t\ times}(X)$. For a random variable X, $\mathbb{E}[X]$ denotes the expected value of X.

II. Problem Setup

A. System Models

Consider the system in Fig. 1. The process is discrete and linear time-invariant and its dynamics is given by

$$x_{k+1} = Ax_k + w_k \tag{1}$$

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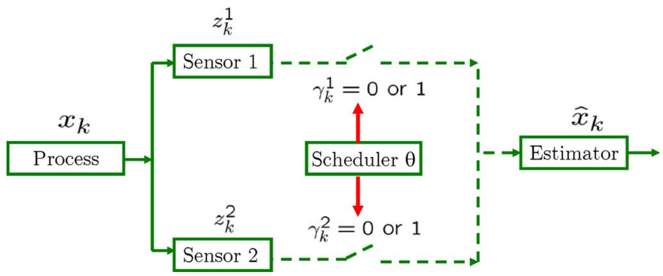

Fig. 1. System block diagram.

where $k \in \mathbb{Z}$ is the time indices, $x_k \in \mathbb{R}^n$ is the process state vector, w_k is the process noise (or disturbance) which is zero-mean white Gaussian with covariance $Q \geq 0$. The initial state x_0 is also a zero-mean Gaussian variable with covariance Π_0. The pair $(A, \sqrt{Q})$ is stabilizable.

Two sensors are used to measure x_k in (1). Their measurement equations are given as follows:

$$y_k^1 = C_1 x_k + v_k^1 \tag{2}$$
$$y_k^2 = C_2 x_k + v_k^2 \tag{3}$$

where $y_k^1 \in \mathbb{R}^{m_1}$ and $y_k^2 \in \mathbb{R}^{m_2}$ are the measurements collected by the two sensors at time k, v_k^1 and v_k^2 are the measurement noises which are zero-mean white Gaussian with covariances $R_1 > 0$ and $R_2 > 0$. Further assume x_0, w_k, v_k^1 and v_k^2 are mutually uncorrelated, and the pair $(A, [C_1' \; C_2']')$ is detectable.

Denote $y_{k_1:k_2}^i = \{y_{k_1}^i, \ldots, y_{k_2}^i\}$ as all the measurements collected by sensor i from time k_1 to k_2. Let D be an integer and define $z_k^i(D)$ as $z_k^i(D) \triangleq y_{k-D+1:k}^i$. Sometimes we write $z_k^i(D)$ as z_k^i when D is clear from the context. Assume sensor i sends z_k^i in a single data packet to a remote estimator.[1] The remote estimator then computes $\hat{x}_k$, the optimal linear estimate of x_k, based on all data received up to time k. Notice that different D renders different data available at the remote estimator. Thus $\hat{x}_k$ and its associated error covariance P_k depend on D implicitly, i.e., they are functions of D.

It is well known that $\hat{x}_k(D)$ and its associated error covariance matrix $P_k(D)$ for the process state in (1) are given by

$$\hat{x}_k(D) = \mathbb{E}[x_k | \text{all data received up to } k] \tag{4}$$
$$P_k(D) = \mathbb{E}[e_k e_k' | \text{all data received up to } k] \tag{5}$$

where $e_k(D) = x_k - \hat{x}_k(D)$ is the estimation error.

Assume at most one sensor can communicate with the remote estimator at each time k due to the limited communication bandwidth. Let γ_k^i be the indicator function whose value (1 or 0) indicates whether sensor i is selected to use the communication channel. Thus the communication bandwidth constraint can be expressed as

$$\gamma_k^1 + \gamma_k^2 \leq 1 \; \forall \, k \geq 1. \tag{6}$$

Let θ be a periodic schedule which assigns values to γ_k^i and Θ be the set of all periodic schedules. Notice that both $\hat{x}_k(D)$ and $P_k(D)$ in (4)–(5) are functions of the schedule θ, thus we sometimes write them as $\hat{x}_k(D, \theta)$ and $P_k(D, \theta)$ to show their dependency on θ explicitly.

[1]This only requires the sensor has some processing capability and a buffer to store the previous $D - 1$ measurement data. Most wireless sensor nodes in the market nowadays have such capability. In typical data networks, a packet is in the order of Kilobytes or even hundreds of Kilobytes. Thus as long as D is reasonably small, one data packet is enough to include $z_k^i(D)$.

B. Problems of Interest

For a given schedule θ with period $N(\theta)$, define $J_i(\theta)$ as the duty cycle of sensor i, i.e.

$$J_i(\theta) = \frac{1}{N(\theta)} \sum_{k=1}^{N(\theta)} \gamma_k^i(\theta) \tag{7}$$

and $P_a(D, \theta)$ as the trace of the average estimation error covariance, i.e.

$$P_a(D, \theta) \triangleq \operatorname{Tr}\left(\lim_{T \to \infty} \frac{1}{T} \sum_{k=1}^{T} P_k(D, \theta) \right). \tag{8}$$

Let $\Psi_i < 1$ be the maximum duty cycle of sensor i. For simplicity, assume Ψ_i is a rational number. The maximum duty cycle is imposed to represent the limited communication energy at the sensor.

In this technical note, we are interested in finding a D and a periodic schedule θ that solves the following optimization problem:[2]

Problem 2.1:

$$\begin{aligned} \min_{D, \theta \in \Theta} \quad & P_a(D, \theta) \\ \text{s.t.} \quad & \gamma_k^1(\theta) + \gamma_k^2(\theta) \leq 1 \\ & J_1(\theta) \leq \Psi_1 \\ & J_2(\theta) \leq \Psi_2 \\ & \Psi_1 + \Psi_2 = 1. \end{aligned}$$

In other words, the optimal D and θ minimize the estimation error at the remote estimator and at the same time satisfies the limited communication energy and bandwidth constraints. As we shall see shortly in Section V

$$\min_{\theta \in \Theta} P_a(\infty, \theta) \leq P_a(D, \theta), \; \forall \, D \geq 1, \; \forall \, \theta \in \Theta.$$

Thus we will first look for the optimal θ^* such that $P_a(\infty, \theta^*)$ is minimum. Then we seek a finite D (as small as possible) such that $P_a(D, \theta^*) = P_a(\infty, \theta^*)$. This approach will be elaborated in the next few sections.

For brevity, we will write $\hat{x}_k(D, \theta)$ as $\hat{x}_k$ (or $\hat{x}_k(\theta)$) and $P_k(D, \theta)$ as P_k (or $P_k(\theta)$), etc., when the underlying D and θ are clear from the context.

III. Optimal Estimation at the Estimator

In this section, we show how to compute $\hat{x}_k$ and P_k based on the Kalman filter [11]. Before we introduce the optimal estimation procedure at the estimator, we first define a few functions to simplify the notations in subsequent sections. Define the functions $h, \tilde{g}_1, \tilde{g}_2$ and $\tilde{g} : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as follows:

$$h(X) \triangleq AXA' + Q \tag{9}$$
$$\tilde{g}_1(X) \triangleq X - XC_1'[C_1XC_1' + R_1]^{-1}C_1X \tag{10}$$
$$\tilde{g}_2(X) \triangleq X - XC_2'[C_2XC_2' + R_2]^{-1}C_2X \tag{11}$$
$$\tilde{g}(X) \triangleq X - XC'[CXC' + R]^{-1}CX \tag{12}$$

where $C = [C_1' \; C_2']')$ and $R = \operatorname{diag}(R_1, R_2)$. Further define the functions g_1, g_2 and and $g : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as

$$g_1 \triangleq \tilde{g}_1 \circ h, \; g_2 \triangleq \tilde{g}_2 \circ h, \; g \triangleq \tilde{g} \circ h.$$

Notice that applying h to P_{k-1} corresponds to the time update step of the Kalman filter, and applying $\tilde{g}_i$ to $h(P_{k-1})$ corresponds to the measurement update step of the Kalman filter if y_k^i is used to improve

[2]In Remark 5.5, we show that with some work the limitation of searching the optimal sequence within the set of periodic sequences can be eliminated.

the estimate. Let I_{k-1} be the set of all measurement data, collected by the sensors at or before $k-1$, available at the estimator. Further let P_{k-1} be the estimation error covariance of the optimal estimate of x_{k-1} given the data set I_{k-1}. Then, one can verify the following equalities:

$$\mathbb{E}[(x_k-\hat{x}_k)(x_k-\hat{x}_k)'|I_{k-1},y_k^1]=g_1(P_{k-1}) \quad (13)$$

$$\mathbb{E}[e_k e_k'|I_{k-1},y_k^2]=g_2(P_{k-1}) \quad (14)$$

$$\mathbb{E}[e_k e_k'|I_{k-1},y_k^1,y_k^2]=g(P_{k-1}). \quad (15)$$

Since $(A,\sqrt{Q})$ is stabilizable and (A,C) is detectable, from standard Kalman filtering analysis [11], the equation $X=g(X)$ has a unique solution $\overline{P}\geq 0$, which is the steady-state error covariance of the Kalman filter when both sensors are used.

The following lemma summarizes a few well-known properties of the functions h, g, and the steady-state error covariance $\overline{P}$. The proofs are omitted.

Lemma 3.1: For any $0\leq X\leq Y$ and any $1\leq t_1\leq t_2$, the following inequalities hold:
1) $h(X)\leq h(Y)$.
2) $g(X)\leq h(X)$.
3) $g(X)\leq g_i(X), i=1,2$.
4) $g_i^{t_1}(\overline{P})\leq g_i^{t_2}(\overline{P}), i=1,2$.

We are now ready to introduce the optimal estimation procedure at the estimator. Let us first consider the simple case when $D=1$. It is easy to see that the optimal estimator is simply the Kalman filter, and the error covariance P_k evolves as

$$P_k=\begin{cases} h(P_{k-1}), & \text{if } \gamma_k^1=\gamma_k^2=0,\\ g_1(P_{k-1}), & \text{if } \gamma_k^1=1,\\ g_2(P_{k-1}), & \text{if } \gamma_k^2=1.\end{cases}$$

When $D\geq 2$, the estimation procedure is different than the Kalman filter. The reason is that at time k, the previously not available data might become available. Thus the previous estimate can be further improved at k. Let $\mathcal{S}_k^t,\ t\leq k$, denote the set of measurements collected at time t that are available at time k. Then $\mathcal{S}_{k_1}^t\subset\mathcal{S}_{k_2}^t,\ \forall\, t\leq k_1\leq k_2$ holds as a newly arrived data packet also contains the previous $D-1$ measurement. The following example illustrates the idea.

Example 1: Assume $D=2$. Consider the following schedule from time 1 to 4. $\gamma_k^1: 0\ 1\ 1\ 0$ and $\gamma_k^2: 1\ 0\ 0\ 1$. Then one can verify that
1) $\mathcal{S}_1^1=\{y_1^2\}$.
2) $\mathcal{S}_2^1=\{y_1^1,y_1^2\},\mathcal{S}_2^2=\{y_2^1\}$.
3) $\mathcal{S}_3^1=\{y_1^1,y_1^2\},\mathcal{S}_3^2=\{y_2^1\},\mathcal{S}_3^3=\{y_3^1\}$.
4) $\mathcal{S}_4^1=\{y_1^1,y_1^2\},\mathcal{S}_4^2=\{y_2^1\},\mathcal{S}_4^3=\{y_3^1,y_3^2\},\mathcal{S}_4^4=\{y_4^2\}$.

Clearly $\mathcal{S}_1^1\subset\mathcal{S}_2^1$ and $\mathcal{S}_3^3\subset\mathcal{S}_4^3$. At time 1, $\hat{x}_1=\mathbb{E}[x_1|\mathcal{S}_1^1]$ and $P_1=g_2(P_0)$. At time 2, y_1^1 becomes available. Thus $\hat{x}_1$ can be further improved, i.e., $\hat{x}_1=\mathbb{E}[x_1|\mathcal{S}_2^1]$ and $P_1=g(P_0)$. Notice that from Lemma 3.1 we indeed have $g(P_0)\leq g_2(P_0)$. Consequently $\hat{x}_2=\mathbb{E}[x_2|\mathcal{S}_2^2]$ and $P_2=g_1\Big(g(P_0)\Big)$. At time 3, the only new information is y_3^1, thus $\hat{x}_2$ remains unchanged and $P_3=g_1(P_2)$. At time 4, $\hat{x}_3$ is first recalculated as y_3^2 becomes available. As a result, $P_4=g_2(g(g_1(g(P_0))))$. The estimation procedure is presented with details in [12]. The key idea is that the newly available data collected at the earliest time, e.g., $t\leq k$, is first used to improve $\hat{x}_t$, which is then used to update $\hat{x}_{t+1}$. This is repeated until we compute $\hat{x}_k$. It is proven in [12] that this procedure leads to the optimal estimate. Furthermore the error covariance matrix P_k can be computed and closed-form expression can be obtained as seen from Example 1.

IV. Necessary Condition

In this section, we present a necessary condition for a schedule to be optimal. This necessary condition is given in the following theorem.

Lemma 4.1: An optimal schedule θ^* to Problem 2.1 satisfies

$$J_1(\theta^*)=\Psi_1,\ J_2(\theta^*)=\Psi_2. \quad (16)$$

Proof: Without loss of generality, consider a feasible schedule θ with $J_1(\theta)<\Psi_1$ and $J_2(\theta)=\Psi_2$. Since $\Psi_1+\Psi_2=1$, we must have $J_1(\theta)+J_2(\theta)<\Psi_1+\Psi_2=1$. Define $\epsilon(\theta)$ and $\delta(\theta)$ as $\epsilon(\theta)\triangleq\Psi_1-J_1(\theta)$ and $\delta(\theta)\triangleq 1-J_1(\theta)-J_2(\theta)$. Then the communication channel is idle (i.e., both sensors are not transmitting) for $\delta(\theta)$ portion of time. Construct a different schedule $\hat{\theta}$ based on θ as follows: *whenever the communication channel is idle under θ, schedule sensor one to send data*. Notice that $J_1(\hat{\theta})=J_1(\theta)+\delta(\theta)=\Psi_1$, thus the new schedule $\hat{\theta}$ is still feasible. From the optimal estimation in Section III, we can write $P_k(\theta)$ as $P_k(\theta)=f_k\circ f_{k-1}\circ\cdots\circ f_1(P_0)$, where

$$f_t=\begin{cases} h, & \text{neither } y_t^1 \text{ nor } y_t^2 \text{ is available},\\ g_1, & \text{only } y_t^1 \text{ is available},\\ g_2, & \text{only } y_t^2 \text{ is available},\\ g, & \text{both } y_t^1 \text{ and } y_t^2 \text{ are available}.\end{cases}$$

Similarly we write $P_k(\hat{\theta})$ as $P_k(\theta)=\hat{f}_k\circ\hat{f}_{k-1}\circ\cdots\circ\hat{f}_1(P_0)$ where $\hat{f}_t$ is defined in the same way as f_t is. Recall that we have defined $\mathcal{S}_k^t(t\leq k)$ as the set of measurements collected at time t that are available at time k. From the construction of $\hat{\theta}$, it is clear that

$$\mathcal{S}_k^t(\theta)\subset\mathcal{S}_k^t(\hat{\theta}). \quad (17)$$

From (17), we immediately arrive at the following four cases:
1) $f_t=h$ and $\hat{f}_t=h$ or g_1 or g_2 or g;
2) $f_t=g_1$ and $\hat{f}_t=g_1$ or g;
3) $f_t=g_2$ and $\hat{f}_t=g_2$ or g;
4) $f_t=g$ and $\hat{f}_t=g$.

In any of the above four cases, we have $\hat{f}_t\leq f_t\ \forall\, t\leq k$, where the inequality is from Lemma 3.1. Therefore we conclude that $P_k(\theta)\geq P_k(\hat{\theta})$. Finally it is trivial to show that $P_k(\theta)\neq P_k(\hat{\theta})$. This shows that θ cannot be optimal. In other words, the optimal θ^* satisfies (16). ■

Lemma 4.1 basically states that in order to achieve minimum error, the available resources have to be fully utilized, which makes intuitive sense. In the next section, we will rely on this necessary condition to construct an optimal schedule.

V. Optimal Periodic Schedule

In this section, we first consider the limiting case $D=\infty$. In other words, when sensor i is scheduled to send a data packet, it sends all the data it has collected up to time k. This may not seem to be practical, however, it has great theoretical values:
1) The estimation error covariance P_k can be proven to be the least among all possible linear filters [13]. An optimal schedule θ^* can be constructed and the resulting $P_a(\infty,\theta^*)$ thus serves as the lower bound of $P_a(D,\theta)$ for any D and any θ.
2) Based on the result obtained for $D=\infty$, we are able to show that the same schedule θ^* satisfies $P_a(D,\theta^*)=P_a(\infty,\theta^*)$ for some finite D.

We start this section by giving an explicit expression of $P_k(\infty,\theta)$, abbreviated as $P_k(\theta)$, for a given schedule θ.

A. Error Covariance Calculation

Consider a periodic schedule θ with period $N(\theta)$. From Lemma 4.1, without loss of generality, we assume that $\gamma_k^1=1$ or $\gamma_k^2=1\ \forall\, k$ and sensor one or two is scheduled at least one time during a period of $N(\theta)$ times. For nonzero γ_k^1 and γ_k^2, define $\tau(\gamma_k^1)$ and $\tau(\gamma_k^2)$ as

$$\tau(\gamma_k^1) \triangleq k - \max\{s : s < k \text{ and } \gamma_s^2 = 1\},$$
$$\tau(\gamma_k^2) \triangleq k - \max\{s : s < k \text{ and } \gamma_s^1 = 1\}.$$

If $\gamma_k^1 = 1$, then the estimator has $\{y_{1:k}^1, y_{1:k-\tau(\gamma_k^1)}^2\}$. Therefore from (13)–(15), the error covariance P_k is given by

$$P_k = \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)' | y_{1:k}^1, y_{1:k-\tau(\gamma_k^1)}^2] = g_1^{\tau(\gamma_k^1)}\left(g^{k-\tau(\gamma_k^1)}(P_0)\right). \quad (18)$$

Similarly, if $\gamma_k^2 = 1$, the error covariance satisfies

$$P_k = g_2^{\tau(\gamma_k^2)}\left(g^{k-\tau(\gamma_k^2)}(P_0)\right). \quad (19)$$

As sensor one or two is scheduled at least once during a period of $N(\theta)$ times, we have $\tau(\gamma_k^i) \le N(\theta), i = 1, 2$. Since the recursion $P_{k+1} = g(P_k)$ converges to $\overline{P}$ exponentially fast for any $P_0 \ge 0$, and in consideration of the fact that $P_a(\theta)$ is defined for infinite time-horizon hence transient contributions of P_k may be neglected, we can assume $g^{k-\tau(\gamma_k^i)}(P_0) = \overline{P}, i = 1, 2$. Consequently, (18)–(19) become

$$P_k = g_1^{\tau(\gamma_k^1)}(\overline{P}) \text{ if } \gamma_k^1 = 1 \quad (20)$$
$$P_k = g_2^{\tau(\gamma_k^2)}(\overline{P}) \text{ if } \gamma_k^2 = 1. \quad (21)$$

The following theorem, which provides a closed-form formula for evaluating $P_a(\theta)$ for a given θ, is a direct consequence of this fact.

Theorem 5.1: For a given periodic schedule θ, $P_a(\theta)$ can be computed as

$$P_a(\theta) = \frac{1}{N(\theta)} \times \mathrm{Tr}\left(\sum_{k=T+1,\gamma_k^1 \neq 0}^{T+N(\theta)} g_1^{\tau(\gamma_k^1)}(\overline{P}) + \sum_{k=T+1,\gamma_k^2 \neq 0}^{T+N(\theta)} g_2^{\tau(\gamma_k^2)}(\overline{P})\right) \quad (22)$$

where T is any integer with $T \ge N(\theta)$.

B. Optimal Periodic Schedule Construction

In this section, we construct an optimal periodic schedule for $D = \infty$. Since Ψ_i is assumed to be a rational number, it can be written as $\Psi_1 = p/q$ for two co-prime integers p and q satisfying $p \le q$. Therefore $\Psi_2 = (q-p)/q$. Without loss of generality assume $p \le (1/2)q$. Let s be an integer ($s \ge 1$) that satisfies

$$(s-1)p < q - 2p \le sp. \quad (23)$$

Notice that such an s always exists. In the special case when $p = (1/2)q$, $s = 0$.

In the following proposition, a periodic schedule θ^* is constructed which is shown to be optimal.

Proposition 5.2: An optimal schedule θ^* can be constructed as follows, where the values of $\gamma_k^2(\theta^*)$ for a single period of q is listed below[3]:

$$\underbrace{(10\underbrace{1\cdots1}_{s\ times})\cdots(10\underbrace{1\cdots1}_{s\ times})}_{q-(s+1)p\ times}\underbrace{(10\underbrace{1\cdots1}_{s-1\ times})\cdots(10\underbrace{1\cdots1}_{s-1\ times})}_{(s+2)p-q\ times}$$

[3]If $s = 0$, $s-1$ times is defined to be zero times.

and $\gamma_k^1(\theta^*)$ is given by $\gamma_k^1(\theta^*) = 1 - \gamma_k^2(\theta^*)\ \forall\, k$. The corresponding $P_a(\theta^*)$ is given by[4]

$$P_a(\theta^*) = \mathrm{Tr}\left(\frac{p}{q}\left(g_1(\overline{P}) + \sum_{j=1}^{s} g_2^j(\overline{P})\right) + \frac{\left(q-(s+1)p\right)}{q} g_2^{s+1}(\overline{P})\right). \quad (24)$$

Proof: We prove the case $s \ge 1$. The proof for $s = 0$ is similar. Notice that $q - (s+1)p + (s+2)p - q = p$ and $[q-(s+1)p](s+1) + [(s+2)p - q]s = q - p$. Thus sensor one is scheduled p times and sensor two is scheduled $q - p$ times within a period of q times exactly. Hence θ^* is feasible. Next from Theorem 5.1, one can verify that $P_a(\theta^*)$ is given by (24).

We now prove the optimality of θ^*. Consider any other schedule θ with period q under which sensor one is scheduled exactly p times[5]. We shall prove θ^* is indeed optimal by showing that

$$P_a(\theta) \ge P_a(\theta^*). \quad (25)$$

Without loss of generality, we assume $g^k(P_0) = \overline{P}$ as the transient contribution of P_k may be neglected. Since $\tau(\gamma_k^1) \ge 1$, from Lemma 3.1 and Theorem 5.1, within a single period of θ

$$\sum_{k,\gamma_k^1(\theta)\neq 0} P_k(\theta) = \sum_{k,\gamma_k^1(\theta)\neq 0} g_1^{\tau(\gamma_k^1)}(\overline{P}) \ge \sum_{k,\gamma_k^1(\theta)\neq 0} g_1(\overline{P}) = p g_1(\overline{P}). \quad (26)$$

Consider all possible values of $\tau(\gamma_k^2)$. Let $\mathcal{B} = \{\tau(\gamma_k^2) : \gamma_k^2 \neq 0\}$. First note that $|\mathcal{B}| = q - p$ as sensor two is scheduled for $q - p$ times. Let b_j $(1 \le j \le q-p)$ be a member of $\mathcal{B}$. Without loss of generality, assume b_j is in ascending order, i.e., $b_1 \le b_2 \cdots \le b_{q-p}$. From its definition, $\tau(\gamma_k^2) = 1$ if and only if sensor two is scheduled to send data at time k and sensor one is scheduled to send data at time $k-1$. Since sensor one is scheduled for p times exactly, we must have $b_p \ge 1$ and $b_{p+1} \ge 2$. Next notice that $\tau(\gamma_k^2) = 2$ if and only if sensor two is scheduled to send data at time k and $k-1$ and sensor one is scheduled to send data at time $k-2$. Again, since sensor one is scheduled for p times exactly, we must have $b_{2p+1} \ge 3$. Following a similar argument, we conclude that $b_{lp+1} \ge l+1,\ 1 \le l \le s$. Therefore we obtain the following inequality:

$$\begin{aligned}
&\sum_{k,\gamma_k^2\neq 0} P_k(\theta) \\
&= \sum_{k,\gamma_k^2\neq 0} g_2^{\tau(\gamma_k^2)}(\overline{P}) = \sum_{j=1}^{q-p} g_2^{b_j}(\overline{P}) \\
&= \sum_{j=1}^{p} g_2^{b_j}(\overline{P}) + \sum_{j=p+1}^{2p} g_2^{b_j}(\overline{P}) + \cdots + \sum_{j=sp+1}^{(s+1)p} g_2^{b_j}(\overline{P}) \\
&\quad + \sum_{j=(s+1)p+1}^{q} g_2^{b_j}(\overline{P}) \\
&\ge \sum_{j=1}^{p} g_2(\overline{P}) + \sum_{j=p+1}^{2p} g_2^2(\overline{P}) + \cdots + \sum_{j=sp+1}^{(s+1)p} g_2^s(\overline{P}) \\
&\quad + \sum_{j=(s+1)p+1}^{q} g_2^{s+1}(\overline{P}) \\
&= p\sum_{j=1}^{s} g_2^j(\overline{P}) + \left(q-(s+1)p\right) g_2^{s+1}(\overline{P}).
\end{aligned}$$

[4]If $s = 0$, $\sum_{j=1}^{s} g_2^j(\overline{P}) \triangleq 0$.

[5]Assuming θ also has period q is without loss of generality. If θ has a different period $\tilde{q}$, then θ^* and θ can be viewed as two periodic schedules with a common period $q\tilde{q}$. From Lemma 4.1, sensor one must be scheduled for $p\tilde{q}$ times within a period of $q\tilde{q}$ times. The proof then extends trivially to cover this case.

Finally we have

$$\begin{aligned} qP_a(\theta) &= \mathrm{Tr}\left(\sum_{k,\gamma_k^1(\theta)\neq 0} P_k(\theta) + \sum_{k,\gamma_k^2(\theta)\neq 0} P_k(\theta)\right) \\ &\geq \mathrm{Tr}\left(pg_1(\overline{P}) + p\sum_{j=1}^{s} g_2^j(\overline{P}) + \Big(q-(s+1)p\Big)g_2^{s+1}(\overline{P})\right) \\ &= qP_a(\theta^*). \end{aligned}$$

The proof is thus complete. ■

Corollary 5.3: Let θ^* be the optimal schedule stated in Proposition 5.2 with period $N = N(\theta^*)$. Consider a schedule θ which also has period N. Then for sufficiently large T, there exists a permutation ϕ on $P_{TN+1}(\theta), \ldots, P_{(T+1)N}(\theta)$ such that

$$P_k(\theta^*) \leq \hat{P}_k(\theta),\ TN+1 \leq k \leq (T+1)N$$

where

$$\Big(\hat{P}_{TN+1}(\theta), \ldots, \hat{P}_{(T+1)N}(\theta)\Big) = \phi\Big(P_{TN+1}(\theta), \ldots, P_{(T+1)N}(\theta)\Big).$$

Remark 5.4: From this corollary, θ^* not only minimizes the average error, but also provides the minimum error at each time (with a time permutation). Therefore θ^* is also optimal if we change the cost function $P_a(\theta)$ to $\max P_k(\theta)$.

Remark 5.5: From the proof of Proposition 5.2, we can eliminate the limitation of searching the optimal sequence within the set of periodic sequences. The idea runs as follows: redefine the cost function $P_a(\theta)$ as $P_a(\theta, T) = (1/T)\sum_{k=1}^{T} P_k(\theta)$. For sufficiently large T (hence the transient period can be ignored), we can prove that the optimal sequence which minimizes $P_a(\theta, T)$ schedules the first sensor (or the second sensor, depending on whether $p \leq (1/2)q$ or $p > (1/2)q$) *as uniformly as possible*. Furthermore, the relative time difference between two adjacent instances that the first (or second) sensor are scheduled remains constant for all sufficiently large T. Therefore, the optimal sequence converges to a periodic sequence over the infinite-time horizon, which can be shown to be exactly the same as that given by Proposition 5.2.

So far we have constructed an optimal schedule θ^* for the limiting case $D = \infty$. The following result is a straightforward extension of Proposition 5.2.

Theorem 5.6: The schedule θ^* constructed in Proposition 5.2 is still optimal for a finite D as long as $D \geq s+2$ where s is given by (23).

Proof: For the θ^* constructed in Proposition 5.2, sensor one is scheduled exactly p times. Thus the maximum time gap between two adjacent scheduling of sensor one is $s+2$. Therefore when sensor one sends a data packet, all its previously collected measurements are available at the estimator side. Thus the case $D \geq s+2$ is equivalent to the limiting case. ■

From (23), $q/p - 2 \leq s < q/p + 1$. Thus θ^* can be constructed if

$$D \geq \frac{q}{p} + 3 = \frac{1}{\Psi_1} + 3. \quad (27)$$

The interpretation of (27) is clear: the number of previous packets to be sent at each time is inversely proportional to its duty cycle.

1) Example 2: Consider the process and sensors (1)–(3) with $A = [0.95\ 0.01; 0\ 0.95]$, $C_1 = [1\ 0]$, $C_2 = [0\ 1]$, $Q = \mathbf{diag}(5,5)$, $R_1 = 2$ and $R_2 = 2$. Let the maximum duty cycle of sensor one and two be $\Psi_1 = 2/7$ and $\Psi_2 = 5/7$. The optimal schedule θ^* is given by Proposition 5.2, and the values of $\gamma_k^i(\theta^*)$ for a single period are γ_k^1 : $0\,1\,0\,0\,1\,0\,0$ and γ_k^2 : $1\,0\,1\,1\,0\,1\,1$. When compare θ^* with the following schedule θ (in a single period of seven times) $\gamma_k^1(\theta)$: $1\,1\,0\,0\,0\,0\,0$ and $\gamma_k^2(\theta)$: $0\,0\,1\,1\,1\,1\,1$, we have the following results.

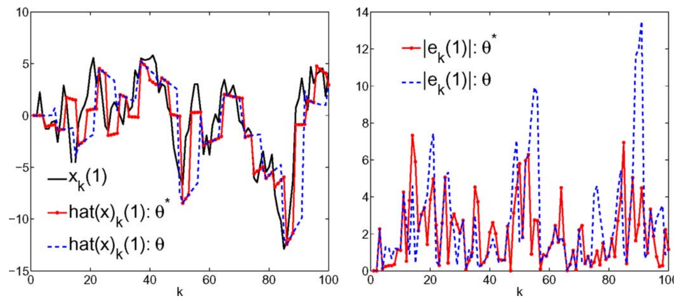

Fig. 2. State estimate and the estimation error: 1st component.

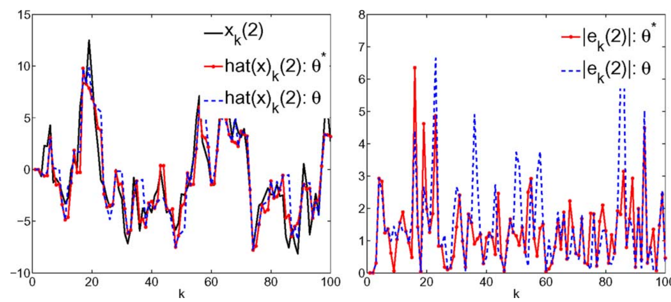

Fig. 3. State estimate and the estimation error: 2nd component.

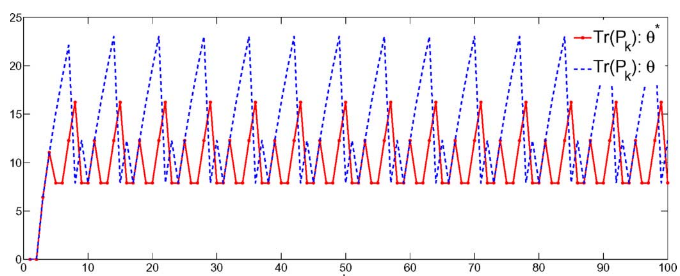

Fig. 4. Trace of the error covariance matrices of the two schedules.

Figs. 2 and 3 show the estimator's performance under the two schedules. We plot the evolution of the two components of x_k, $\hat{x}_k(\theta^*)$, $\hat{x}_k(\theta)$ and their corresponding estimation errors. From the right half of these two figures, we see that θ^* indeed provides smaller estimation error on the average. Fig. 4 plots the evolution of the trace of the corresponding error covariance matrices. The simulation results agree well with the theories developed in the previous sections.

VI. Conclusion

In this technical note, we consider a sensor scheduling problem. Two sensors with limited energy budget are to be scheduled over a finite bandwidth communication network. We first derive a necessary condition for a schedule to be optimal. Based on this necessary condition, we construct an optimal schedule that minimizes the error covariance of the state estimate and at the same time satisfies the constraints on the limited sensor energy and communication bandwidth. Examples and simulation verify the results developed in the technical note.

Extending the result in this note to multiple sensor scenario will be pursued in the future. The condition $D \geq s+2$ as stated in Theorem 5.6 is only sufficient to guarantee the cost function $P_a(D, \theta)$ can achieve its minimum, thus Theorem 5.6 can be viewed as providing an achievable lower bound of $P_a(D, \theta)$ under the assumption that the sensor can send $s+2$ most recent data. An interesting problem is to consider how does the cost function degrade when D decreases. In this case, quantization/coding issues will come into play. There exists a wealth body of literature addressing quantization/coding issues. Combining these issues with scheduling is worthwhile to study.

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On Solving Optimal Policies for Finite-Stage Event-Based Optimization

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***Abstract*—Event-based optimization (EBO) has been developed to model a specific type of problems, in which decisions can be made only when certain events occur. Because the event sequence usually is not Markovian, how to solve optimal policies for EBOs remains open in general. Motivated by real applications, we focus on finite-stage EBOs with discrete state space in this technical note and make two contributions. First, we show that this EBO can be converted to a partially observable Markov decision process (POMDP). Based on this connection, existing exact and approximate solution methodologies for POMDPs can then be applied to EBOs. Second, we develop the performance difference and derivative formulas and the potential-based policy iteration algorithm, which converges to the global optimum. This algorithm is then applied to a node activation problem in wireless sensor network.**

***Index Terms*—Discrete event dynamic systems (DEDS's), event-based optimization (EBO), partially observable Markov decision process (POMDP).**

I. INTRODUCTION

The dynamics of many systems such as transportation system, manufacturing system, and electric power grid follow not only physical laws but also man-made rules. These systems are known as discrete event dynamic systems (DEDS's) [1], where state transitions are triggered by events. Event-based optimization (EBO) has been developed [2] to model a specific type of problems, in which decisions can be made only when certain events occur. However, it is known that the event sequence usually is not Markovian. Thus how to solve the optimal policies for EBOs remains open in general.

Though most existing studies on EBO focus on infinite-stage problems, finite-stage EBO captures the essence of many applications, too. For example, consider a wireless sensor network (WSN) that is deployed in an area of interest (AoI) to monitor some objects. When an area is monitored by multiple sensors in the same time, the object in that area is detected with higher probability, and the sensors receive a higher reward. The batteries of the sensors can be recharged and have limited cycle lifetimes. The central node can only activate fully-charged sensors. In order to save the wireless communication power, an activation decision is made only when some sensors just become fully charged or discharged, i.e., when ready or sleep events occur. The question is how to activate the sensors to maximize the total reward in the lifetime of the network. As will be shown in Section V-B, this problem can be modeled as a finite-stage EBO.

We focus on finite-stage EBOs with discrete state and action spaces and make two contributions in this technical note. First, we show that the EBO can be converted to a partially observable Markov decision

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