Automatica 47 (2011) 1693–1698

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Sensor data scheduling for optimal state estimation with communication energy constraint ☆

Ling Shi [a,*], Peng Cheng [b], Jiming Chen [b]

[a] Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

[b] Institute of Industrial Process Control, Department of Control Science and Engineering, Zhejiang University, Hang Zhou, China

ARTICLE INFO

Article history:
Received 2 December 2009
Received in revised form
30 December 2010
Accepted 25 January 2011
Available online 17 March 2011

Keywords:
Kalman filtering
Sensor scheduling
Estimation theory
Networked control

ABSTRACT

In this paper, we consider sensor data scheduling with communication energy constraint. A sensor has to decide whether to send its data to a remote estimator or not due to the limited available communication energy. We construct effective sensor data scheduling schemes that minimize the estimation error and satisfy the energy constraint. Two scenarios are studied: the sensor has sufficient computation capability and the sensor has limited computation capability. For the first scenario, we are able to construct the optimal scheduling scheme. For the second scenario, we are able to provide lower and upper bounds of the minimum error and construct a scheduling scheme whose estimation error falls within the bounds.

1. Introduction

Networked sensing and control systems have gained much interest in the past decade (Hespanha, Naghshtabrizi, & Xu, 2007). Applications of networked sensing and control systems are found in a growing number of areas, including autonomous vehicles, environment and habitat monitoring, industrial automation, transportation, etc.

In some networked sensing applications, sensors are battery-powered, hence only limited energy is available for data collection and transmission. Consequently a sensor cannot transmit its measurement data at all times due to the energy constraint, and it has to decide whether to send its current data packet or not. This decision-making process is referred to as *sensor data scheduling*.

On one extreme, sending no data consumes no energy. However, without receiving and processing the sensor measurement data, the estimation error of the underlying parameters may grow rapidly which is undesirable in situations such as target tracking and rescue and surveillance. On the other extreme, sending data at all times assures that the estimation error is a minimum but at the price of high energy cost. The latter case may not even be feasible due to the energy constraint. Thus proper schedule of the sensor data transmission is needed such that the energy constraint is satisfied and the estimation error is kept as small as possible. Constructing such a proper sensor data scheduling scheme is the focus of this paper.

Sensor scheduling has been a hot topic of research for many years. Different formulations and approaches have been proposed. Baras and Bensoussan (1988) studied nonlinear state estimation problem and considered scheduling a set of sensors so as to optimally estimate a function of an underlying parameter. Walsh and Ye (2001) and Walsh, Ye, and Bushnell (2002) studied the problem of when to schedule which process to access to the network so that all processes remain stable. Gupta, Chung, Hassibi, and Murray (2006) considered a different scheduling problem where there is one process and multiple sensors. They proposed a stochastic sensor scheduling scheme and provided the optimal probability distribution over the sensors to be selected. Tiwari, Jun, Jeffcoat, and Murray (2005) studied the problem of sensor scheduling for discrete-time state estimation using a Kalman filter. They considered two processes and one sensor and proposed schemes to determine which process that the sensor needs to observe in order to minimize the total estimation error. Shi, Epstein, Sinopoli, and Murray (2007) combined the ideas from Gupta et al. (2006) and Tiwari et al. (2005) and proposed two novel scheduling schemes in a sensor network by employing feedback from the estimator to the sensors. Hovareshti, Gupta, and Baras (2007) considered sensor scheduling using smart sensors, i.e., sensors with some memory and processing capabilities, and demonstrated that estimation

☆ The work by L. Shi is partially supported by HKUST direct allocation grant DAG11EG06G. The work by P. Cheng and J. Chen is partially supported by NSFC grant no. 60974122. The material in this paper was partially presented at the 8th World Congress on Intelligent Control and Automation, July 7–9, 2010, Jinan, Shandong, China. This paper was recommended for publication in revised form by Associate Editor Giuseppe De Nicolao under the direction of Editor Ian R. Petersen.

* Corresponding author. Tel.: +852 2358 7055; fax: +852 2358 1485.
E-mail addresses: eesling@ust.hk (L. Shi), pcheng@iipc.zju.edu.cn (P. Cheng), jmchen@iipc.zju.edu.cn (J. Chen).

0005-1098/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2011.02.037

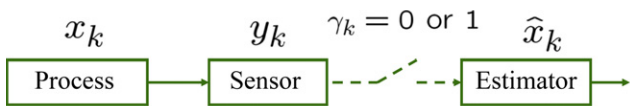

Fig. 1. System block diagram.

performance can be improved. Sandberg, Rabi, Skoglund, and Johansson (2008) considered estimation over a heterogeneous sensor network. Two types of sensors were investigated: the first type has low-quality measurement but small processing delay, while the second type has high-quality measurement but large processing delay. Using a time-periodic Kalman filter, they showed how to find an optimal schedule of the sensor communication. Similar work has been done by Arai, Iwatani, and Hashimoto (2008, 2009) where fast sensor scheduling was proposed for networked sensor systems. Savage and Scala (2009) considered the problem of optimal sensor scheduling for scalar systems that minimizes the terminal error.

The main contributions of this paper and comparison with existing work from the literature are summarized as follows.

(1) We develop sensor scheduling schemes that provide the best estimation quality subject to sensor energy constraint. To the best of our knowledge, the problem formulation is novel.
(2) We focus on scheduling of the sensor measurement data, while most of the existing work focused on scheduling of a set of (heterogeneous) sensors.
(3) Since the solution space contains infinite scheduling schemes which are discrete in nature, most existing work proposed algorithms that typically generate a suboptimal schedule, and nothing in general is said on the optimality of the proposed schedule. However, in this paper, when the sensor has sufficient computation capability, we are able to construct an optimal scheduling scheme; when the sensor has limited computation capability, we are able to provide a lower and upper bound of the estimation error for the optimal scheme.

The remaining of the paper is organized as follows. In Section 2, we introduce the system models and problem setup. In Section 3, we define some frequently used notations and provide some preliminaries on the Kalman filter. In Section 4, we provide the necessary condition for optimal scheduling schemes. In Section 5, we study the scenario when the sensor has sufficient computation and present an optimal scheduling scheme. In Section 6, we study the scenario when the sensor has limited computation and present a suboptimal schedule. Concluding remarks are given in the end.

Notations. $\mathbb{Z}$ is the set of non-negative integers. k is the time index. $\mathbb{N}$ is the set of natural numbers. $\mathbb{R}^n$ in n-dimensional Euclidian space. $\mathbb{S}^n_+$ is the set of n by n positive semi-definite matrices. When $X \in \mathbb{S}^n_+$, we simply write $X \geq 0$; when X is positive definite, we write $X > 0$. For functions $f, f_1, f_2 : \mathbb{S}^n_+ \to \mathbb{S}^n_+$, $f_1 \circ f_2$ is defined as $f_1 \circ f_2(X) \triangleq f_1(f_2(X))$ and f^t is defined as $f^t(X) \triangleq \underbrace{f \circ f \circ \cdots \circ f}_{t \text{ times}}(X)$.

2. Problem setup

2.1. System models

Consider the following discrete linear time-invariant system (Fig. 1)

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

$$y_k = Cx_k + v_k. \tag{2}$$

In the above equations, $x_k \in \mathbb{R}^n$ represents the current state of the process, $y_k \in \mathbb{R}^m$ is the measurement data taken by the sensor at time k, $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are zero-mean Gaussian random noises with covariances $\mathbb{E}[w_k w_j'] = \delta_{kj} Q \geq 0$, $\mathbb{E}[v_k v_j'] = \delta_{kj} R > 0$, $\mathbb{E}[w_k v_j'] = 0\ \forall j, k$, where $\delta_{kj} = 0$ if $k \neq j$ and $\delta_{kj} = 1$ otherwise. The initial state x_0 is also a zero-mean Gaussian random vector that is uncorrelated with w_k or v_k and has covariance $\Pi_0 \geq 0$. Further assume that $(A, \sqrt{Q})$ is controllable and (C, A) is observable.

Assume that the sensor communicates its data packet with a remote estimator via a network. Let

$$Y_k = \{y_1, \ldots, y_k\} \tag{3}$$

be all the measurements collected by the sensor from time 1 to k. The sensor's local state estimate $\hat{x}^s_k$ and its corresponding error covariance P^s_k are calculated as

$$\hat{x}^s_k = \mathbb{E}[x_k | Y_k], \tag{4}$$

$$P^s_k = \mathbb{E}[(x_k - x^s_k)\,(x_k - x^s_k)' | Y_k]. \tag{5}$$

Most commercially available sensor nodes nowadays have different transmission power levels (Xiao, Cui, Luo, & Goldsmith, 2006). Reliable data flow is typically achieved using high power transmission. Low power transmission may cause unreliable data flow and data packet drops are typical consequences. For simplicity, we assume the sensor operates in two energy levels. When the sensor uses a high energy Δ at time k, the data packet can be successfully delivered to the remote estimator; when the sensor uses a low energy δ, the data packet can be successfully delivered only with probability $\lambda \in (0, 1)$. We assume both Δ and δ are rational numbers. When δ energy is used, let $\lambda_k = 1$ or 0 be the indicator function whether the data packet arrives at the estimator successfully or not. Assume λ_k's are i.i.d Bernoulli random variables and $\mathbb{E}[\lambda_k] = \lambda$.

Let $\gamma_k = 1$ or 0 be the sensor's decision variable at time k whether it should send its current data packet using Δ or δ energy. Let θ denote a scheduling scheme that defines the value of γ_k at each k. Clearly the set of all scheduling schemes consists of 2^k different schemes up to time k, most of which are unstructured and are intractable to analyze. We thus focus on the subset of all periodic scheduling schemes which we denote as Θ.

Denote $D_k(\theta)$ as the set of all data packets received by the estimator up to time k. In general $D_k(\theta)$ could be different from Y_k defined in Eq. (3) due to the possible data packet drops. Similar to calculating $\hat{x}^s_k$ and P^s_k, for a given θ, the state estimate $\hat{x}_k(\theta)$ and its associated error covariance $P_k(\theta)$ at the remote state estimator are calculated as

$$\hat{x}_k(\theta) = \mathbb{E}[x_k | D_k(\theta)], \tag{6}$$

$$P_k(\theta) = \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)' | D_k(\theta)]. \tag{7}$$

For simplicity, we shall write $\hat{x}_k(\theta)$ as $\hat{x}_k$, etc., when the underlying scheduling scheme θ is clear from the context.

2.2. Problems of interest

For a given θ, define $J(\theta)$ as the average energy cost associated with it, i.e.,

$$J(\theta) \triangleq \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} (\gamma_k \Delta + (1 - \gamma_k)\delta), \tag{8}$$

and $P_a(\theta)$ as the average expected estimation error covariance, i.e.,

$$P_a(\theta) \triangleq \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \mathbb{E}[P_k]. \tag{9}$$

Let Ψ be a given energy budget. Assume that Ψ is a rational number and $\delta \leq \Psi \leq \Delta$.

In this paper, we are interested in finding a periodic scheduling scheme θ that solves the following optimization problem.

Problem 2.1.

$$\min_{\theta\in\Theta} P_a(\theta)$$
$$\text{s.t.}\quad J(\theta) \le \Psi.$$

In other words, we wish to seek a scheduling scheme $\theta \in \Theta$ such that the communication energy constraint is satisfied at the sensor side and the average estimation error is minimum at the estimator side. A scheme θ is *feasible* if $J(\theta) \le \Psi$. A scheme θ^* is *optimal* if it is feasible and $P_a(\theta^*) \le P_a(\theta)$ for any other feasible θ.

We consider two scenarios in this paper. In the first scenario, the sensor has sufficient computation and memory. It runs a local Kalman filter to compute $\hat{x}_k^s$ and P_k^s, and sends $\hat{x}_k^s$ to the remote estimator. We call this *estimate communication*. In the second scenario, the sensor has limited computation and memory, and only sends y_k (or a few previous measurements packed together with y_k if C is not invertible) to the remote estimator. We call this *measurement communication*.

3. Definitions and Kalman filter preliminaries

To facilitate the analysis in subsequent sections, we first present some notations as well as a brief summary of the standard Kalman filter. The following terms that are frequently used in subsequent sections are defined. It is assumed that (A, C, Q, R) are the same as they appear in Section 2.1. We define the function h and $g : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as

$$h(X) \triangleq AXA' + Q, \tag{10}$$

$$g(X) \triangleq X - XC'[CXC' + R]^{-1}CX. \tag{11}$$

It is straightforward to verify that if $0 \le X \le Y$, then $h(X) \le h(Y)$, $g(X) \le g(Y)$ and $g(X) \le X$ (e.g., Lemma A.1 in Shi, Epstein, and Murray (2010)). In particular, $g \circ h \le h$.

3.1. Sensor with limited computation

Assume $\gamma_k = 1\,\forall\, k \ge 1$. It is well known that $\hat{x}_k$ and P_k in Eqs. (6) and (7) can be calculated alternatively using a Kalman filter (**KF**). We write $(\hat{x}_k, P_k)$ in compact form as $(\hat{x}_k, P_k) = \mathbf{KF}(\hat{x}_{k-1}, P_{k-1}, y_k)$, which represents the following set of recursive equations:

$$\begin{cases} \hat{x}_k^- = A\hat{x}_{k-1}, \\ P_k^- = AP_{k-1}A' + Q, \\ K_k = P_k^- C'[CP_k^- C' + R]^{-1}, \\ \hat{x}_k = A\hat{x}_{k-1} + K_k(y_k - CA\hat{x}_{k-1}), \\ P_k = (I - K_kC)P_k^-. \end{cases}$$

The recursion starts from $\hat{x}_0 = 0$ and $P_0 = \Pi_0$. Since y_k is always available at the sensor side, if the sensor has sufficient computation capability, $\hat{x}_k^s$ and P_k^s in Eqs. (4) and (5) are also calculated following the same procedure, i.e., $(\hat{x}_k^s, P_k^s) = \mathbf{KF}(\hat{x}_{k-1}^s, P_{k-1}^s, y_k)$. With some manipulation, P_k can be shown to satisfy

$$P_k = g \circ h(P_{k-1}). \tag{12}$$

Denote $\overline{P}$ as the steady-state error covariance, i.e., $\overline{P}$ is the unique positive semi-definite solution[1] to $g \circ h(X) = X$. The following lemma about $\overline{P}$ is useful in establishing some results in the next few sections. The proof can be found in the Appendix.

Lemma 3.1. *If $1 \le t_1 \le t_2$, then $h^{t_1}(\overline{P}) \le h^{t_2}(\overline{P})$. Furthermore, $h(\overline{P}) \ne \overline{P}$.*

If $\gamma_k = 0$, i.e., y_k is not available at the estimator, then it can be shown that the optimal estimate $\hat{x}_k$ simply equals $A\hat{x}_{k-1}$. The corresponding error covariance matrix P_k is given by

$$P_k = h(P_{k-1}).$$

3.2. Sensor with sufficient computation

When the sensor has sufficient computation capability, it runs a local Kalman filter to compute $\hat{x}_k^s$ and sends $\hat{x}_k^s$ to the remote estimator. It is straightforward to show that the optimal state estimate and error covariance at the estimator side are computed as

$$(\hat{x}_k, P_k) = \begin{cases} (A\hat{x}_{k-1}, h(P_{k-1})), & \text{if } \gamma_k = 0, \\ (\hat{x}_k^s, P_k^s), & \text{if } \gamma_k = 1. \end{cases}$$

For any $P_0^s \ge 0$, P_k^s converges to $\overline{P}$ exponentially fast, therefore without loss of generality, we assume that the Kalman filter enters steady-state at the sensor side. Then P_k is simply given by

$$P_k = \begin{cases} h(P_{k-1}), & \text{if } \gamma_k = 0 \text{ and } \lambda_k = 0, \\ \overline{P}, & \text{otherwise.} \end{cases}$$

4. Necessary condition for optimal scheduling schemes

In this section, we present a necessary condition for an optimal scheduling scheme. We will then use this condition to construct an optimal scheme.

For a given θ, define π_1 as the fraction of times that the sensor uses high energy Δ, i.e.,

$$\pi_1(\theta) \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \gamma_k(\theta). \tag{13}$$

Similarly, define π_2 as the fraction of times that the sensor uses low energy δ, i.e., $\pi_2 = 1 - \pi_1$. Then we have the following necessary condition of an optimal scheme θ^*.

Theorem 4.1. *Let θ^* be an optimal scheme to Problem 2.1. Then*

$$\pi_1(\theta^*) = \frac{\Psi - \delta}{\Delta - \delta}. \tag{14}$$

Proof. First consider the estimate communication. For simplicity, we write $\pi_1(\theta^*) = \pi_1^*$. Notice that $J(\theta)$ in Eq. (8) can be written as

$$J(\theta) = \frac{1}{N} \lim_{N\to\infty} \sum_{k=1}^{N} (\gamma_k\Delta + (1 - \gamma_k)\delta) = \pi_1\Delta + \pi_2\delta.$$

Therefore if π_1^* is given by Eq. (14), then

$$J(\theta^*) = \pi_1^*\Delta + \pi_2^*\delta = \Psi.$$

Hence θ^* is feasible. Now consider a different scheme θ with

$$\pi_1 < \frac{\Psi - \delta}{\Delta - \delta}.$$

We construct a scheme $\hat{\theta}$ according to the following two steps.

(1) Set $\hat{\theta} = \theta$.

(2) For arbitrary $\frac{\pi_1^* - \pi_1}{2\pi_2}$ fraction of times in $\hat{\theta}$ that use δ energy, change to use Δ energy.

Notice that the sensor uses δ energy for π_2 fraction of times in θ. Thus after step (2), the sensor uses Δ energy for $\frac{1}{2}(\pi_1 + \pi_1^*)$ fraction of times in $\hat{\theta}$. In other words

$$\hat{\pi}_1 = \frac{1}{2}(\pi_1 + \pi_1^*) \quad \text{and} \quad \hat{\pi}_2 = \frac{1}{2}(\pi_2 + \pi_2^*).$$

Therefore

$$\begin{aligned} J(\hat{\theta}) &= \frac{1}{2}(\pi_1 + \pi_1^*)\Delta + \frac{1}{2}(\pi_2 + \pi_2^*)\delta \\ &= \frac{1}{2}(\pi_1^*\Delta + \pi_2^*\delta) + \frac{1}{2}(\pi_1\Delta + \pi_2\delta) < \Psi, \end{aligned}$$

i.e., $\hat{\theta}$ is feasible. Consider any $N \ge 1$. For $k \le N$, if $\gamma_k(\theta) = 1$, then $\gamma_k(\hat{\theta}) = 1$, hence $P_k(\hat{\theta}) = \overline{P} = P_k(\theta)$; if $\gamma_k(\theta) = 0$, then $\gamma_k(\hat{\theta}) = 1$

[1] Since (C, A) is observable and $(A, \sqrt{Q})$ is controllable, from standard Kalman filtering analysis (Anderson & Moore, 1979), $\overline{P} \ge 0$ exists and is unique.

or 0. From Lemma 3.1, it is straightforward to show that

$$\mathbb{E}[P_k(\hat{\theta})] \le \mathbb{E}[P_k(\theta)] \quad \text{and} \quad \mathbb{E}[P_k(\theta)] \ne \mathbb{E}[P_k(\hat{\theta})].$$

Consequently,

$$\sum_{k=1}^{N} \mathbb{E}[P_k(\hat{\theta})] \le \sum_{k=1}^{N} \mathbb{E}[P_k(\theta)]$$

and

$$\sum_{k=1}^{N} \mathbb{E}[P_k(\theta)] \ne \sum_{k=1}^{N} \mathbb{E}[P_k(\hat{\theta})].$$

As the above two inequalities hold for any $N \ge 1$, we conclude that $P_a(\hat{\theta}) \le P_a(\theta)$ and $P_a(\theta) \ne P_a(\hat{\theta})$, i.e., θ cannot be optimal. Therefore π_1^* corresponding to any optimal scheme θ^* has to satisfy Eq. (14).

The proof for the measurement communication scenario follows similarly. □

5. Optimal scheduling scheme for estimate communication

In this section, we consider the estimate communication scenario. We will construct an optimal scheduling scheme based on the necessary condition in Theorem 4.1. Since the cost function is over an infinite horizon, without loss of generality, we assume that the Kalman filter at the sensor side has entered steady state and we ignore its transient dynamics.

Let θ_T be a periodic scheme with period T which is defined as

$$\gamma_k(\theta_T) \triangleq \begin{cases} 1, & \text{if } k \bmod T = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Define $\epsilon_T \triangleq \mathbb{E}[P_T(\theta_T)]$, which has the following property.

Lemma 5.1. *$\epsilon_T \le \epsilon_{T+1}$ for any $T \ge 1$.*

Proof. First we have $\epsilon_1 = \overline{P} \le \lambda\overline{P} + (1-\lambda)h(\overline{P}) = \epsilon_2$. For $T \ge 2$, with some manipulation, one can verify that

$$\epsilon_T = \lambda\overline{P} + (1-\lambda)^{T-1}h^{T-1}(\overline{P}) + \sum_{i=1}^{T-2} \lambda[(1-\lambda)^i h^i(\overline{P})]. \tag{15}$$

Therefore

$$\begin{aligned} \epsilon_{T+1} - \epsilon_T &= (1-\lambda)^T h^T(\overline{P}) + \lambda(1-\lambda)^{T-1}h^{T-1}(\overline{P}) \\ &\quad - (1-\lambda)^{T-1}h^{T-1}(\overline{P}) \\ &= (1-\lambda)^T h^T(\overline{P}) - (1-\lambda)^T h^{T-1}(\overline{P}) \ge 0. \quad \square \end{aligned}$$

We are now ready to present an optimal scheduling scheme for the estimate communication scenario. Since Δ, δ, and Ψ are all rational numbers, π_1^* given by Eq. (14) is also a rational number. Therefore we can write π_1^* as $\pi_1^* = \frac{p}{q}$ for two co-prime integers $p \le q$.

Theorem 5.2. *Consider the estimate communication scenario.*

(1) $p < \frac{1}{2}q$: *Let z be the largest integer such that $z \le \frac{q}{p}$. An optimal scheduling scheme θ^* can be constructed in terms of the values of $\gamma_k(\theta^*)$ over a period q as follows:*

$$\underbrace{(1\ \underbrace{0\cdots0}_{z-1\text{ times}})\cdots(1\ \underbrace{0\cdots0}_{z-1\text{ times}})}_{p(z+1)-q\text{ times}}\ \underbrace{(1\underbrace{0\cdots0}_{z\text{ times}})\cdots(1\underbrace{0\cdots0}_{z\text{ times}})}_{q-pz\text{ times}}$$

The average estimation error $P_a(\theta^*)$ is given by

$$P_a(\theta^*) = \frac{1}{q}\left[p\sum_{i=1}^{z}\epsilon_i + (q-pz)\epsilon_{z+1}\right]$$

where $\epsilon_1 = \overline{P}$ and ϵ_i is given by Eq. (15) for $i \ge 2$.

(2) $p \ge \frac{1}{2}q$: *An optimal scheduling scheme θ^* can be constructed in terms of the values of $\gamma_k(\theta^*)$ over a period q as follows:*

$$\underbrace{(1\ 0)\cdots(1\ 0)}_{q-p\text{ times}}\ \underbrace{(1)\cdots(1)}_{2p-q\text{ times}}$$

The average estimation error $P_a(\theta^*)$ is given by

$$P_a(\theta^*) = \frac{1}{q}\left[p\epsilon_1 + (q-p)\epsilon_2\right].$$

Proof. Without loss of generality, we compare θ^* with an arbitrary schedule θ which has the same period q and under which Δ energy is scheduled exactly p times over a period.[2] Since the cost function is taken over an infinite-horizon, to prove $P_a(\theta^*) \le P_a(\theta)$, it is sufficient to prove that

$$\sum_{k=k_1^*}^{k_1^*+q-1} P_k(\theta^*) \le \sum_{k=k_1}^{k_1+q-1} P_k(\theta)$$

where k_1^* and k_1 are any two times such that $\gamma_{k_1^*}(\theta^*) = 1$ and $\gamma_{k_1}(\theta) = 1$. Since $k_1^* = 1$, without loss of generality, we can assume $k_1 = 1$. Thus we need to prove that

$$\sum_{k=1}^{q} P_k(\theta^*) \le \sum_{k=1}^{q} P_k(\theta).$$

Denote b_α as the number of 0's which appears α time steps after a 1 in the set $\{\gamma_k(\theta) : k = 1, \ldots, q\}$. Since there are exact p 1's in $\{\gamma_k(\theta) : k = 1, \ldots, q\}$, there are at most p 0's in $\{\gamma_k(\theta) : k = 1, \ldots, q\}$ that are right following a 1. Therefore $b_1 \le p$. Similarly since there are b_1 0's that are right following a 1, there are at most b_1 0's that are right following such a 0. This shows that $b_2 \le b_1$. Continuing this argument, we arrive at $b_{\alpha+1} \le b_\alpha \le \cdots \le b_1 \le p$ for any $\alpha \in \mathbb{N}$.

(1) $p < \frac{1}{2}q$: Since $\frac{q}{p} > 2$, from the definition of z, we conclude $z \ge 2$. From the construction of θ^*, we have

$$\sum_{k=1}^{q} P_k(\theta^*) = p\epsilon_1 + p\epsilon_2 + \cdots + p\epsilon_z + (q-pz)\epsilon_{z+1}.$$

We also have, from the definition of b_α, the following equality:

$$\sum_{k=1}^{q} P_k(\theta) = p\epsilon_1 + b_1\epsilon_2 + b_2\epsilon_3 + \cdots + b_{q-p}\epsilon_{q-p+1}$$

with the constraint that $b_1 + b_2 + \cdots + b_{q-p} = q - p$. Therefore

$$\begin{aligned} &\sum_{k=1}^{q} P_k(\theta) - \sum_{k=1}^{q} P_k(\theta^*) \\ &= (b_1 - p)\epsilon_2 + (b_2 - p)\epsilon_3 + \cdots + (b_{z-1} - p)\epsilon_z \\ &\quad - (q-pz)\epsilon_{z+1} + b_z\epsilon_{z+1} + \cdots b_{q-p}\epsilon_{q-p+1} \\ &\ge (b_1 - p)\epsilon_{z+1} + (b_2 - p)\epsilon_{z+1} + \cdots + (b_{z-1} - p)\epsilon_{z+1} \\ &\quad - (q-pz)\epsilon_{z+1} + b_z\epsilon_{z+1} + \cdots b_{q-p}\epsilon_{z+1} \\ &= [b_1 + b_2 + \cdots + b_{q-p} - (q-p)]\epsilon_{z+1} = 0, \end{aligned}$$

where the inequality is from Lemma 5.1 as well as the fact that $b_\alpha \le p$ for any $\alpha \in \mathbb{N}$.

[2] First, if θ has a different period $\tilde{q}$, then we can consider that θ^* and θ have a common period $q\tilde{q}$ and Δ energy is scheduled $p\tilde{q}$ times over a period. Second, if Δ energy is scheduled for less than p times under θ, then from Theorem 4.1, there always exists a $\hat{\theta}$ under which Δ energy is scheduled exactly p times and $P_a(\hat{\theta}) \le P_a(\theta)$.

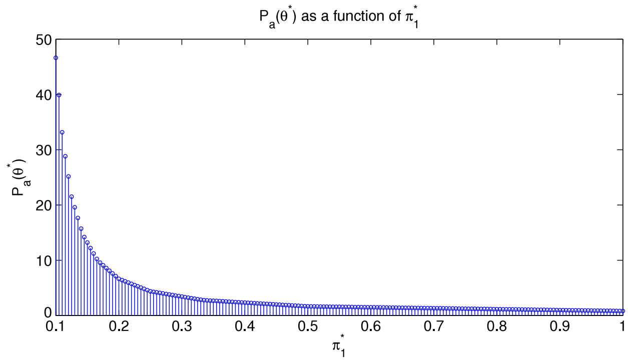

Fig. 2. Larger π_1^* leads to smaller $P_a(\theta^*)$.

(2) $p \geq \frac{1}{2}q$: In this case,

$$\sum_{k=1}^{q} P_k(\theta^*) = p\epsilon_1 + (q-p)\epsilon_2,$$

$$\sum_{k=1}^{q} P_k(\theta) = p\epsilon_1 + b_1\epsilon_2 + b_2\epsilon_3 + \cdots + b_{q-p}\epsilon_{q-p+1}.$$

The rest of the proof is similar to that of the case $p < \frac{1}{2}q$.

The proof is thus complete. □

Remark 5.3. Theorem 5.2 states that under an optimal schedule, the instances when Δ energy is scheduled should be separated *as uniformly as possible*. When the sensor has multiple energy levels to choose from, which is typical in most commercial sensors, scheduling which energy level to use at what time is much more complicated and challenging, especially when different power levels introduce different packet drop rates. It is out of the scope of this paper to address this more interesting and practical issue, which will be pursued in future work.

Example 5.4. Consider $\pi_1^* = \frac{3}{10}$. Then θ^* has period 10, $z = 3$, and $\gamma_k(\theta^*)$ for $1 \leq k \leq 10$ is given by $\{1, 0, 0, 1, 0, 0, 1, 0, 0, 0\}$. When $\pi_1^* = \frac{7}{10}$, θ^* has period 10 and $\gamma_k(\theta^*)$ for $1 \leq k \leq 10$ is given by $\{1, 0, 1, 0, 1, 0, 1, 1, 1, 1\}$. We also plot $P_a(\theta^*)$ as a function of π_1^* in Fig. 2 when π_1^* varies from $\frac{1}{10}$ to 1. The system parameters are $A = 2,\ C = 1,\ Q = 1$ and $R = 1$. As seen from the figure, larger π_1^*, i.e., more energy budget, leads to smaller $P_a(\theta^*)$, which makes intuitive sense.

6. Error bounds for measurement communication

In this section, we consider the measurement communication scenario. In particular, we consider the case that the measurement matrix C is invertible. As shown in Shi et al. (2010), by sending a few previous measurements together with y_k, similar results can be obtained.

Define $\overline{M} \triangleq C^{-1}RC^{-1'}$. The following lemmas are from Shi et al. (2010).

Lemma 6.1. *For any $k \geq 1$, if $\gamma_k = 1$ or $\gamma_k = 0$ with $\lambda_k = 1$, then $P_k \leq \overline{M}$.*

Lemma 6.2. *Assume $P_0 \geq \overline{P}$. Then for all $k \geq 0$, $P_k \geq \overline{P}$.*

Define $\hat{\epsilon}_T$ as follows. $\hat{\epsilon}_1 \triangleq \overline{M}$. For $T \geq 2$,

$$\hat{\epsilon}_T \triangleq \lambda\overline{M} + (1-\lambda)^{T-1}h^{T-1}(\overline{M}) + \sum_{i=1}^{T-2}\lambda[(1-\lambda)^i h^i(\overline{M})]. \quad (16)$$

Finding the optimal scheduling scheme for measurement communication is more difficult than for estimate communication. Nevertheless, by applying the optimal scheme θ^* in Theorem 5.2 to the measurement communication, we obtain the following result.

Theorem 6.3. *Assume $\pi_1^* = \frac{p}{q}$ for two co-prime integers $p \leq q$.*

1. *$p \leq \frac{1}{2}q$: $P_a(\theta^*)$ is bounded below by*

$$P_a(\theta^*) \geq \frac{1}{q}\left[p\sum_{i=1}^{z}\epsilon_i + (q-pz)\epsilon_{z+1}\right]$$

and $P_a(\theta^*)$ is bounded above by

$$P_a(\theta^*) \leq \frac{1}{q}\left[p\sum_{i=1}^{z}\hat{\epsilon}_i + (q-pz)\hat{\epsilon}_{z+1}\right].$$

2. *$p > \frac{1}{2}q$: $P_a(\theta^*)$ is bounded by*

$$\frac{1}{q}[p\epsilon_1 + (q-p)\epsilon_2] \leq P_a(\theta^*) \leq \frac{1}{q}[p\hat{\epsilon}_1 + (q-p)\hat{\epsilon}_2].$$

Proof. Direct result from Theorem 5.2, Lemmas 6.1 and 6.2. □

From Theorem 6.3, when $\overline{P}$ and $\overline{M}$ are close, we can approximate the optimal scheduling for the measurement communication scenario by the one obtained for the estimate communication scenario (as in Theorem 5.2).

7. Conclusion

In this paper, we investigate the sensor data scheduling problem. Two scenarios are studied. In the first scenario, we assume that the sensor has sufficient computation and it runs a Kalman filter and sends the estimate to a remote estimator. For this scenario, we are able to construct a scheduling scheme that minimizes the estimation error at the estimator yet satisfies the sensor's communication energy constraint. In the second scenario, we assume that the sensor has limited computation and sends the measurement data to the remote estimator. For this scenario, we are able to construct lower and upper bounds of the minimum estimation error.

There are many interesting future directions along the line of this work: find tighter bounds for the measurement communication scenario; investigate the problem of multiple energy level scheduling; close the control loop and schedule the control data; study the dual problem, i.e., finding a scheduling scheme that minimizes the communication energy yet guarantees a desired estimation quality.

Acknowledgements

The authors would like to thank the associate editor and the reviewer for their comments and suggestions which help improve the presentation of the paper.

Appendix. Supporting lemmas

Lemma A.1. (1) *Let $X > 0$. If $CXC' = 0$, then $C = 0$.*
(2) *Let $\mathcal{O}$ be an m by n matrix ($m \geq n$) with full column rank and $Q \geq 0$ be an n by n matrix. If $\mathcal{O}Q\mathcal{O}' = 0$, then $Q = 0$.*

Proof. (1) Let $Y = [Y_{ij}] = CX^{\frac{1}{2}}$. Then

$$\sum_{i,j} Y_{ij}^2 = \mathrm{Tr}(YY') = 0$$

implies $Y = 0$. Therefore $C = YX^{-\frac{1}{2}} = 0$. (2) Assume $Q \neq 0$. Since $Q \geq 0$, it must have an eigenvalue $\beta > 0$. Let $x \neq 0$ be an eigenvector of Q corresponding to β, i.e., $Qx = \beta x$. Since $\mathcal{O}$ has full column rank, there exists y such that $\mathcal{O}'y' = x$. We then obtain the following contradiction $0 = y\mathcal{O}Q\mathcal{O}'y' = x'Qx = \beta x'x \neq 0$. □

Proof to Lemma 3.1. First notice that $\overline{P} = g \circ h(\overline{P}) \leq h(\overline{P})$. Therefore by applying h on both sides of the inequality we get $\overline{P} \leq h(\overline{P}) \leq h^2(\overline{P})$. By repeating the same procedure, we obtain

$$\overline{P} \leq h(\overline{P}) \leq \cdots \leq h^{t-1}(\overline{P}) \leq h^t(\overline{P}) \quad \forall t \geq 0.$$

Next assume

$$\overline{P} = h(\overline{P}) \tag{A.1}$$

Since $\overline{P} = g \circ h(\overline{P})$, we get

$$h(\overline{P})C'[Ch(\overline{P})C' + R]^{-1}Ch(\overline{P}) = 0.$$

As $[Ch(\overline{P})C' + R]^{-1} > 0$, from part one of Lemma A.1,

$$h(\overline{P})C' = 0.$$

From Eq. (A.1), $h^t(\overline{P})C' = 0 \; \forall \, t \geq 0$. Consider $t = n$. Then

$$0 = Ch^n(\overline{P})C' = CA^n\overline{P}A'^nC' + \sum_{i=0}^{n-1} CA^iQA'^iC'.$$

As $CA^n\overline{P}A'^nC' \geq 0$ and $CA^iQA'^iC' \geq 0$ for all i, we conclude

$$0 = \sum_{i=0}^{n-1} \mathrm{Tr}(CA^iQA'^iC') = \mathrm{Tr}(\mathcal{O}Q\mathcal{O}')$$

where $\mathcal{O} = [C' \; A'C' \; \cdots \; A'^{n-1}C']'$. Since $\mathcal{O}Q\mathcal{O}' \geq 0$, all eigenvalues of $\mathcal{O}Q\mathcal{O}'$ must be zero. Consequently, $\mathcal{O}Q\mathcal{O}' = 0$. As (C, A) is observable, $\mathcal{O}$ has full column rank, from part two of Lemma A.1, $\mathcal{O}Q\mathcal{O}' = 0$ leads to the fact that $Q = 0$, which violates the assumption that $(A, \sqrt{Q})$ is controllable. The proof is thus complete. □

References

Anderson, B., & Moore, J. (1979). *Optimal filtering*. Prentice Hall.

Arai, S., Iwatani, Y., & Hashimoto, K. (2008). Fast and optimal sensor scheduling for networked sensor systems. In *Proceedings of the IEEE conference on decision and control. Cancun, Mexico*.

Arai, S., Iwatani, Y., & Hashimoto, K. (2009). Fast sensor scheduling for spatially distributed heterogeneous sensors. In *Proceedings of the American control conference. St. Louis, MO, USA*.

Baras, J. S., & Bensoussan, A. (1988). Sensor scheduling problems. In *Proceedings of the IEEE conference on decision and control. Austin, TX, USA*.

Gupta, V., Chung, T., Hassibi, B., & Murray, R. M. (2006). On a stochastic sensor selection algorithm with applications in sensor scheduling and dynamic sensor coverage. *Automatica*, *42*(2), 251–260.

Hespanha, J. P., Naghshtabrizi, P., & Xu, Y. (2007). A survey of recent results in networked control systems. *Proceedings of the IEEE*, *95*(1), 138–162.

Hovareshti, P., Gupta, V., & Baras, J. S. (2007). Sensor scheduling using smart sensors. In *Proceedings of the IEEE conference on decision and control. New Orleans, LA, USA*.

Sandberg, H., Rabi, M., Skoglund, M., & Johansson, K. H. (2008). Estimation over heterogeneous sensor networks. In *Proceedings of the IEEE conference on decision and control. Cancun, Mexico*.

Savage, C. O., & Scala, B. F. L. (2009). Optimal scheduling of scalar gauss-markov systems with a terminal cost function. *IEEE Transactions on Automatic Control*, *54*(May), 1100–1105.

Shi, L., Epstein, M., & Murray, R. M. (2010). Kalman filtering over a packet-dropping network: a probabilistic perspective. *IEEE Transactions on Automatic Control*, *55*(9), 594–604.

Shi, L., Epstein, M., Sinopoli, B., & Murray, R. M. (2007). Effective sensor scheduling schemes in a sensor network by employing feedback in the communication loop. In *Proceedings of the IEEE conference on control applications*. Singapore.

Tiwari, A., Jun, M., Jeffcoat, D. E., & Murray, R. M. (2005). Analysis of dynamic sensor coverage problem using kalman filters for estimation. In *Proceedings of the 16th IFAC world congress*.

Walsh, G. C., & Ye, H. (2001). Scheduling of networked control systems. *IEEE Transactions on Control Systems Magazine*, *21*(February), 57–65.

Walsh, G. C., Ye, H., & Bushnell, L. G. (2002). Stability analysis of networked control systems. *IEEE Transactions on Control Systems Technology*, *10*(May), 438–446.

Xiao, J.-J., Cui, S., Luo, Z.-Q., & Goldsmith, A. J. (2006). Power scheduling of universal decentralized estimation in sensor networks. *IEEE Transactions on Signal Processing*, *54*(2), 413–422.

Ling Shi received his B.S. degree in Electrical and Electronic Engineering from the Hong Kong University of Science and Technology in 2002 and Ph.D. degree in Control and Dynamical Systems from California Institute of Technology in 2008. He is currently an Assistant Professor at the Department of Electronic and Computer Engineering at the Hong Kong University of Science and Technology. His research interests include networked control systems, wireless sensor networks and distributed control.

Peng Cheng received the B.E. degree in Automation, and the Ph.D. degree in Control Science and Engineering in 2004 and 2009 respectively, both from Zhejiang University, Hangzhou, PR China. He is currently working as a Postdoctoral researcher in the State Key Laboratory of Industrial Control Technology, Zhejiang University. His research interests include networked estimate and control, robust control and nonlinear systems.

Jiming Chen received B.Sc. degree and Ph.D. degree both in Control Science and Engineering from Zhejiang University in 2000 and 2005, respectively. He was a visiting researcher at INRIA in 2006, National University of Singapore in 2007, and University of Waterloo from 2008 to 2010. Currently, he is a full professor with Department of control science and engineering, and the coordinator of group of Networked Sensing and Control in the State Key laboratory of Industrial Control Technology at Zhejiang University, China. His research interests are estimation and control over sensor network, sensor and actuator network, target tracking in sensor networks, optimization in mobile sensor network. He currently serves associate editors for several international Journals, e.g., Wireless Communication and Mobile Computing (Wiley). He also serves as a Cochair for Ad hoc and Sensor Network Symposium, IEEE Globecom 2011, IEEE MASS 2011 Publicity CoChair, IEEE DCOSS 2011 Publicity CoChair, etc., and TPC member for IEEE ICDCS 2010, IEEE MASS 2010, IEEE INFOCOM 2011, etc.